

ON THE NO-GRAVITY LIMIT OF GRAVITY

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We argue that Relative Locality may arise in the no gravity $G \rightarrow 0$ limit of gravity. In this limit gravity becomes a topological field theory of the BF type that, after coupling to particles, may effectively deform its dynamics. We briefly discuss another no gravity limit with a self dual ground state as well as the topological ultra strong $G \rightarrow \infty$ one.

Keywords: Quantum Gravity Phenomenology; Relative Locality; Topological field theories.

It was rather clear from the very first days of the Quantum Gravity Phenomenology research program¹ that there might exist a class of potentially observable phenomena of the quantum gravitational origin exhibiting themselves in the form of minute deviations from the standard special relativistic kinematics and dynamics. This includes the models with Lorentz Invariance Violation as well as the theories, in which the relativistic symmetries are still present, but become deformed. Both such models can be described in the framework of Relative Locality² and are generically characterized by the observation that the deviations from special relativity could be associated with the presence of a nontrivial geometry of momentum space.

Quantum Gravity is a regime, in which both quantum and gravitational phenomena are ‘strong’ i.e., the characteristic length scale of the process in question is of order of Planck length $\ell_P = \sqrt{\hbar G}$ and the characteristic energy scale is of order of Planck mass $M_P = \sqrt{\hbar/G}$. One can, however, consider a class of processes, in which the characteristic length scale is much larger than ℓ_P so that one can safely neglect the effects caused by spacetime foamy structure, but the energies are still close to Planckian.

The presence of a scale is a prerequisite necessary for the emergence of a nontrivial geometry. In the case of theories with the scale of mass it is the momentum space that, in contrast with special relativity, may acquire a nontrivial geometry. This may result, in turn, in the emergence of new phenomena² that might be detectable in present experiments or in the foreseeable future.

There are two ways one can get to the Relative Locality limit of Quantum Gravity. The first, more direct one, is to consider highly energetic processes with large characteristic length scales like the ultra-Planckian scattering with the impact parameter much larger than ℓ_P .^{3,4} Here we take the second, more indirect way, considering the $G \rightarrow 0$, or no-gravity, limit of the classical general relativity (see⁵ for early discussion.) In this limit both gravitational and quantum effects are negligibly small and one could look for the Relative Locality regime, if present.

The no-gravity limit can be most directly taken in the framework, in which gravity is constructed as a constrained topological BF theory.^{6,7} The starting point

of this approach is the BF topological field theory action with the (anti) de Sitter gauge group ($SO(3, 2)$ or $SO(4, 1)$, respectively) supplemented with the term that breaks this symmetry down to the local Lorentz one

$$S = \frac{1}{16\pi} \int_{\mathcal{M}} B^{IJ} \wedge F_{IJ}(A) - \frac{\beta}{2} B^{IJ} \wedge B_{IJ} - \frac{\alpha}{4} \epsilon_{ijkl} B^{ij} \wedge B^{kl}, \quad (1)$$

where I, J, \dots are the (anti) de Sitter Lie algebra indices and i, j, \dots are the ones of its Lorentz sub-algebra.

It should be mentioned that there is a natural coupling of gravity defined by the action (1) with point particles, which, as in the case of 2+1 gravity, are represented by Wilson lines.⁸

Solving the algebraic field equations for the field B^{IJ} , decomposing the connection A^{IJ} into the Lorentz and translational parts

$$A^{ij} = \omega^{ij}, \quad A^{i4} = \frac{1}{\ell_c} e^i \quad (2)$$

and plugging the result into the action (1) one gets the Holst action of gravity with cosmological constant $\Lambda = 3/\ell_c^2$ appended by a number of topological terms (a linear combination of Pontryagin, Euler, and Nieh-Yan classes)

$$32\pi G S = \int R^{ij} \wedge e^k \wedge e^l \epsilon_{ijkl} + \frac{\Lambda}{6} \int e^i \wedge e^j \wedge e^k \wedge e^l \epsilon_{ijkl} + \frac{2}{\gamma} \int R^{ij} \wedge e_i \wedge e_j. \quad (3)$$

The physical coupling constants, the Newton's constant G and the Immirzi-Barbero parameter γ are related to the coupling constants of the original action (1) as follows

$$G = \frac{\alpha^2 + \beta^2}{\alpha} \frac{1}{\Lambda}, \quad \gamma = \frac{\beta}{\alpha}. \quad (4)$$

There are two interesting limits of the theory described by (1) leading to the no-gravity regime. The first is to fix an arbitrary value of the Immirzi-Barbero parameter $\gamma \neq i$ (i.e., to assume the fixed relation $\beta = \gamma \alpha$) and then to take the limit $\alpha \rightarrow 0$. In this case we get from (1) a pure BF topological field theory, which is in many respects similar to gravity in 2+1 dimensions. It is well known that in 2+1 dimensions gravity is topological. Moreover, after coupling to particles and solving for (topological) degrees of freedom of gravity one obtains, both classically^{9,10} and quantum mechanically¹¹ a deformed particle mechanics with curved momentum space. It is hoped that the no-gravity limit of the physical 3+1 dimensional case an analogous deformation of the dynamics of particles coupled gravity arises.

Another possibility is to take the Immirzi-Barbero parameter $\gamma \rightarrow i$ which translates to $\beta \rightarrow \pm i\alpha$ while keeping α arbitrary. In this case we have to do with the (anti) self dual limit of the action (1), which reminds the setup discussed by Smolin.⁵ It would be very interesting to analyze the perturbation theory (classical and quantum) around this self dual ground state in powers of $\delta\alpha = \alpha \pm i\beta$ given that, as stressed in,⁷ such perturbation theory is going to be, by construction, manifestly diffeomorphism invariant. However, it is not clear if the self dual ground state, when coupled to particles is going to exhibit any kind of the Relative Locality behavior.

Last, but not least one should mention the third possible limit that can be read off from (4). If we fix the value of β and then go with α to zero we obtain again a topological theory (called sometime, misleadingly in the present context, the BF theory with the cosmological constant β), which corresponds this time to the ultra strong limit of gravity $G \rightarrow \infty$. There are many indications (see¹² for a recent review) that in this limit spacetime becomes effectively lower dimensional. As discussed in¹³ there are good reasons to believe that also in this limit gravity coupled to particles may show signs of the Relative Locality behavior, but the full understanding of this requires further studies.

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References

1. G. Amelino-Camelia, “Are we at the dawn of quantum gravity phenomenology?,” *Lect. Notes Phys.* **541** (2000) 1 [gr-qc/9910089].
2. G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin, *Phys. Rev. D* **84** (2011) 084010 [arXiv:1101.0931 [hep-th]].
3. H. L. Verlinde and E. P. Verlinde, “Scattering at Planckian energies,” *Nucl. Phys. B* **371** (1992) 246 [hep-th/9110017].
4. D. Amati, M. Ciafaloni and G. Veneziano, “Classical and Quantum Gravity Effects from Planckian Energy Superstring Collisions,” *Int. J. Mod. Phys. A* **3** (1988) 1615.
5. L. Smolin, “The G(Newton) \rightarrow 0 limit of Euclidean quantum gravity,” *Class. Quant. Grav.* **9** (1992) 883 [hep-th/9202076].
6. L. Smolin, “A Holographic formulation of quantum general relativity,” *Phys. Rev. D* **61** (2000) 084007 [hep-th/9808191].
7. L. Freidel and A. Starodubtsev, “Quantum gravity in terms of topological observables,” hep-th/0501191.
8. L. Freidel, J. Kowalski-Glikman and A. Starodubtsev, “Particles as Wilson lines of gravitational field,” *Phys. Rev. D* **74** (2006) 084002 [gr-qc/0607014].
9. H. J. Matschull and M. Welling, “Quantum mechanics of a point particle in 2+1 dimensional gravity,” *Class. Quant. Grav.* **15**, 2981 (1998) [arXiv:gr-qc/9708054].
10. C. Meusburger and B. J. Schroers, “Phase space structure of Chern-Simons theory with a non-standard puncture,” *Nucl. Phys. B* **738** (2006) 425 [hep-th/0505143].
11. L. Freidel and E. R. Livine, “Effective 3d quantum gravity and non-commutative quantum field theory,” *Phys. Rev. Lett.* **96**, 221301 (2006) [arXiv:hep-th/0512113].
12. S. Carlip, “Spontaneous Dimensional Reduction in Short-Distance Quantum Gravity?,” arXiv:0909.3329 [gr-qc].
13. J. Kowalski-Glikman and A. Starodubtsev, “Effective particle kinematics from Quantum Gravity,” *Phys. Rev. D* **78** (2008) 084039 [arXiv:0808.2613 [gr-qc]].